Reply to "Comment on 'Calculation of electromagnetic field components for a fundamental Gaussian beam' "

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(Received 9 January 2007; published 13 March 2007)

Using a multivariable series method, we calculated the eighth-order expression of the vector potential of a fundamental Gaussian beam, thereby correcting the mistake in our previous paper [Phys. Rev. E **72**, 046501 (2005)].

DOI: 10.1103/PhysRevE.75.038502 PACS number(s): 41.75.Jv, 42.55.Ah, 41.20.Jb, 42.68.Ay

The fundamental Gaussian beam is generally available and is often used in theoretical simulations of its interaction with charged particles. The expressions of the vector potential for a fundamental Gaussian beam was obtained by perturbatively solving the Holmholtz equation order by order [1–5], which is a quite complicated procedure. In our previous work [5] the sixth-order expression of the vector potential is correct, but the eighth-order expression was arrived at incorrectly. This fact was point out by Luo and Liu [6] who did not provide an effective way to calculate the vector potential of fundamental Gaussian beam. The latter is realized in this Comment.

The equation obeyed by the vector potential for a fundamental Gaussian beam is

$$\nabla^2\mathbf{A} + k^2\mathbf{A} = 0. \tag{1}$$

By assuming that the vector potential is polarized in the x direction and changing the coordinates (x,y,z) to ($\xi=x/r_0$, $\eta=y/r_0$, $\zeta=z/l$, where r_0 is the waist radius and $l=kr_0^2$ the diffraction length), the vector potential can be written as

$$\mathbf{A} = \psi(\xi,\eta,\zeta)e^{-i\zeta/s^2}\hat{x}, \tag{2}$$

where $s=1/kr_0$ is the perturbation parameter. Equation (1) can be recast as

$$\left(\frac{\partial^2}{\partial\xi^2} + \frac{\partial^2}{\partial\eta^2} - 2i\frac{\partial}{\partial\zeta} + s^2\frac{\partial^2}{\partial\zeta^2}\right)\psi = 0. \tag{3}$$

For a fundamental Gaussian beam, the solution is independent of the azimuth angle. So in the cylindrical polar coordinate frame, Eq. (3) becomes

$$\left(\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} - 2i\frac{\partial}{\partial\zeta} + s^2\frac{\partial^2}{\partial\zeta^2}\right)\psi = 0, \tag{4}$$

where $\rho^2=\xi^2+\eta^2$. We substitute the variables ρ and ζ by $q=1/(i+2\zeta)$ and $\chi=\rho^2 q$. Setting $\psi=f(q,\chi)e^{-i\chi}$, Eq. (4) becomes

$$\begin{aligned}(s^2\chi^2 q + \chi)f_{\chi\chi} + s^2q^3 f_{qq} + 2s^2\chi q^2 f_{\chi q} + (1 - i\chi + 2s^2\chi q \\ - 2is^2\chi^2 q)f_\chi + (iq + 2s^2q^2 - 2is^2\chi q^2 f_q - (i + s^2\chi^2 q \\ + 2is^2\chi q)f = 0,\end{aligned} \tag{5}$$

where the subscripts q and χ denote derivatives. We look for a series solution for $f(q,\chi)$ of the form

$$f(q,\chi) = \sum_{m,n=0}^{\infty} a_{m,n}\chi^m q^n = \sum_{n=0}^{\infty} q^n \sum_{m=0}^{\infty} a_{m,n}\chi^m = \sum_{n=0}^{\infty} q^n f_n(\chi), \tag{6}$$

which is just the perturbation solution of the fundamental Gaussian beam. By straightforward calculation, Eq. (5) is transformed into equations of $a_{m,n}$:

$$\begin{aligned}s^2\{[(m+1)(m+2)a_{m+2,n} - 2i(m+1)a_{m+1,n} - a_{m,n}]\chi^{m+2}q^{n+1} \\ + [2(m+1)(n+1)a_{m+1,n+1} - 2i(n+1)a_{m,n+1}]\chi^{m+1}q^{n+2} \\ + [2(m+1)a_{m+1,n} - 2ia_{m,n}]\chi^{m+1}q^{n+1} + (n+1)(n \\ + 2)a_{m,n+2}\chi^m q^{n+3} + 2(n+1)a_{m,n+1}\chi^m q^{n+2}\} + \{[(m+1)(m \\ + 2)a_{m+2,n} - i(m+1)a_{m+1,n}]\chi^{m+1}q^n + i(n \\ + 1)a_{m,n+1}\chi^m q^{n+1} + [(m+1)a_{m+1,n} - ia_{m,n}]\chi^m q^n\} = 0.\end{aligned} \tag{7}$$

The recursion relation for $a_{m,n}$ is

$$\begin{aligned}\chi^m q^n : (m+1)^2 a_{m+1,n} - i(m-n+1)a_{m,n} + s^2(m+n-1)(m \\ + n)a_{m,n-1} - 2is^2(m+n-1)a_{m-1,n-1} - s^2 a_{m-2,n-1} = 0.\end{aligned} \tag{8}$$

There are two initial independent coefficients a_{00} and a_{01}. Because a_{00} corresponds to a constant solution of Eq. (1), we set it to zero. Equation (8) includes five different coefficients and fails to constitute a self-contained recursion formula. Two rules (a) $a_{1n}=0$ and (b) $a_{m,n}=0$ as $m>2n$ [5], together with two independent initial coefficients, make all coefficients $a_{m,n}$ be calculable through Eq. (8). The sixth and eighth orders are

$$f_6 = q^3 s^6\left(-24i - 12i\chi^2 + 8\chi^3 + 3i\chi^4 + 2\chi^5 - \frac{i}{6}\chi^6\right)$$

and

$$f_8 = q^4 s^8\left(120 + 60\chi^2 + 40i\chi^3 - 15\chi^4 - 4i\chi^5 - \frac{37}{6}\chi^6 + i\chi^7 + \frac{1}{24}\chi^8\right).$$

At last, we point out from Eq. (8) any order expression of the vector potential for a fundamental Gaussian beam and the corresponding electromagnetic field components can be calculated by a computer program, and the whole solution always diverges.

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